# Lab 3: The Calculus of Inverse Functions

This lab uses *Mathematica* to invert functions and to use their derivatives to integrate. It also develops the hyperbolic functions and their inverses. These are described in Section 6.11 in our textbook. Also see pages 54-55 in the guidebook by Crooke and Ratcliffe.

## Step 1

Before you access the Lab3 notebook, be sure that you have both the single house icon for your own account and the double-house icon for the **math212** account on the shelf of your File Viewer. You should also have subdirectories named **labs**, **notebooks**, and **reports** in your account, and your lab files,[1] your notebooks, and your lab reports should all be filed away in these subdirectories. In short, your directory should look like this:

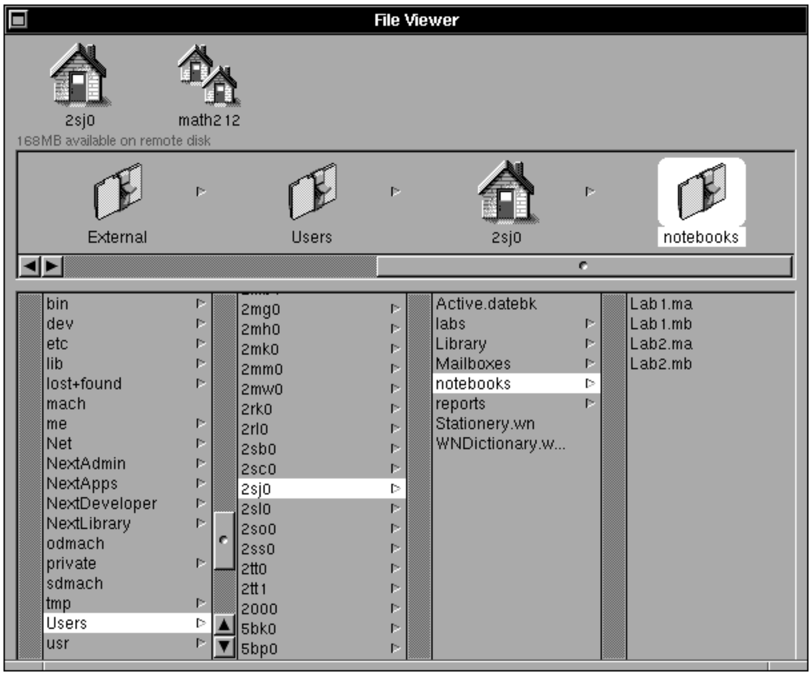


Figure 1 Your File Viewer should look like this

If it doesn't, work through Steps 1-2 of Lab 2 now to make it so.

## Step 2

Now copy the **Lab3.ma** and **Lab3.mb** notebook files into your own **notebooks** subdirectory, just as you did with the **Lab2.ma** and **Lab2.mb** files last week. If you don't remember how to do this, review Step 3 of Lab 2.[2]

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[1] You can get a copy of each lab handount (including this one) from the **math212/labs** directory.
[2] Or ask someone.

## Step 3

Now open your copy of the **Lab3.ma** notebook[3] and double-click on the main cell rectangle to view the 4 main sections of this lab.

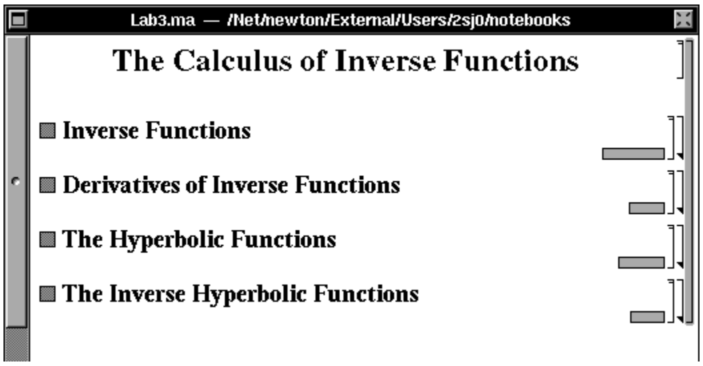


Figure 2 The main sections of **Lab3**

Begin working through this lab, just as you did Lab1 and Lab2. Be sure to *open all subcells* and *execute all Mathematica commands* in the order in which they appear.

## Step 4

In the first section, note that the assignment operator that defines the inverse function `g[y_]` is a simple equals sign (**=**) instead of the usual colon-equals combination (**:=**). Also note that in the rule definitions,[4] the underline (**_**) is required on the left side of the rule, just as it is in function definitions.

The explanation at the end of the second section is based upon the derivation illustrated in Fig. 2. The conclusion is that the slopes of the tangent lines of inverse functions are reciprocals. The diagram shows the 45-degree line intersecting the two tangent lines which have inclinations $\alpha$ and $\beta$. Since the slope of a line is equal to the tangent of its inclination angle, it follows from the derivation that the slopes are reciprocals.[5]

## Step 5

After you and your lab partner have finished working through the notebook, prepare to write your own lab reports describing what you have learned.[6] Use the *WriteNow* word processor to do this, just as you did in Lab1 and Lab2. Refer to Step 5 of Lab2. Be sure to illustrate your work with examples.[7]

Don't forget to save your work regularly. Use Command-s.[8]

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[3] Double-click on the **Lab3.ma** listing in your **labs** subdirectory.
[4] e.g. **`sqrt = Sqrt[u_^2] -> Abs[u]`**
[5] Note that reciprocal slopes are not the same as negative reciprocal slopes, which means that the lines are perpendicular.
[6] You should study the "sample" lab reports for Lab1 and Lab2 in the **math212/reports** directory.
[7] Note that the examples used in the "sample" lab reports are all taken from the notebook exercises.
[8] Hold one of the green Command keys down and press the S key.

In any triangle, the sum of any two interior angles is equal to the other exterior angle. Therefore, $\beta + \theta = 45^\circ$ and $45^\circ + \theta = \alpha$. Then, since $\tan 45^\circ = 1$:

$$\begin{aligned}
\tan\alpha &= \tan(45^\circ + \theta) \\
&= \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ\ \tan\theta} \\
&= \frac{1 + \tan\theta}{1 - \tan\theta} \\
\tan\beta &= \tan(45^\circ - \theta) \\
&= \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ\ \tan\theta} \\
&= \frac{1 - \tan\theta}{1 + \tan\theta}
\end{aligned}$$

Thus, $\tan\beta = 1/\tan\alpha$.

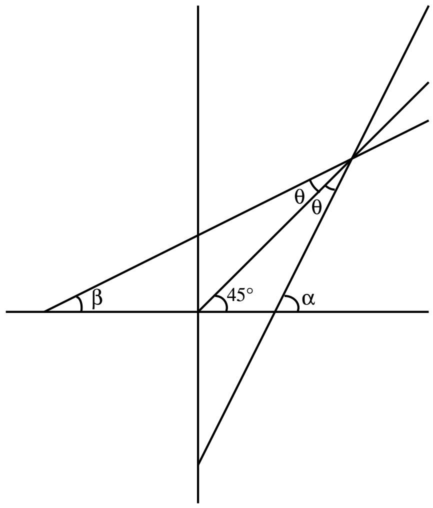


Figure 2 The derivation that inverse functions have reciprocal derivatives

## Step 6

Here are some ideas to consider when writing your report:

### Inverse Functions

1. How can *Mathematica* be used to invert a function?
2. How can *Mathematica* determine analytically[9] that two functions are inverses?
3. How can *Mathematica* determine graphically that two functions are inverses?
4. What new things can you teach to *Mathematica*?

### Derivatives of Inverse Functions

1. How are the derivatives of inverse functions related?
2. How can *Mathematica* verify analytically that an expression is the derivative of the inverse of another function?
3. How can *Mathematica* verify graphically that an expression is the derivative of the inverse of another function?

### The Hyperbolic Functions

1. How are the hyperbolic functions similar to the circular functions?
2. How are they different?
3. How can *Mathematica* verify an identity analytically?

### The Inverse Hyperbolic Functions

1. How are these functions useful in calculus?
2. How can *Mathematica* verify the formulas in the exercises?

[9] i.e. using only the rules of algebra and calculus

## Step 7

After you finish writing your Lab report, mail it to **math212** just as you did for Lab 1. Be sure that the *WriteNow* icon for your report is in the Send window before you click on the Deliver button.

## Appendix: *Mathematica* Commands

Here are some of the *Mathematica* commands that are used in this lab:

| | |
|---|---|
| `Solve[ y==f[x], x ]` | Solves the equation `y=f(x)` for `x` in terms of `y`. |
| `%[[1]]` | Extract the first item from the list in the previous output. |
| `g[y_] = x /. %[[1]]` | Defines g(y) to be the expression that x assigned to in the first item from the list in the previous output. |
| `sqrtsq = Sqrt[u_^2] -> Abs[u]` | Defines the rule **`sqrtsq`** that any expression of the form `Sqrt[u^2]` may be replaced by `Abs[u]`. |
| `g[f[x]] /. sqrtsq` | Uses the rule **`sqrtsq`** to evaluate `g[f[x]]`. |
| `f[g[y]] //. {multlg,lgex}` | Uses the rules **`mulg`** and **`lgex`** <u>repeatedly</u> to evaluate `g[f[x]]`. |
| `AspectRatio -> Automatic` | Forces the scales on the $x$-axis and $y$-axis to be the same. |
| `// Simplify` | Simplifies the current output. |